Phys 410
Spring 2013
Lecture #3 Summary
28 January, 2013

We considered projectile motion with air resistance. The drag force is directed opposite to the instantaneous velocity and depends on the speed as $\vec{f} = -f(v)\hat{v}$, where $f(v) = 0 + bv + cv^2$, and $\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{\vec{v}}{v}$ is the direction unit vector of the velocity. The linear term bv in $f(v)$ arises from viscous friction as the object moves through the fluid (air, water, honey, etc.), and depends on the linear dimension of the object, D. The quadratic term cv^2 arises from accelerating the fluid out of the way of the object, and depends on the cross-sectional area of the object, D^2. The ratio of the two forces is $\frac{f_{quad}}{f_{lin}} = 1.56 \times 10^3 \frac{s}{m^2} Dv$ for a spherical object of diameter D moving through air at STP (20^{o}C, 1 atmosphere pressure). We found that the quadratic force dominates the linear force for a free-falling skydiver, and a golf ball flying through the air. The two forces are comparable for a falling rain-drop, and the linear force dominates for a small oil particle falling in air in the Millikan oil drop experiment.

An object falling under the influence of gravity and the linear drag force obeys the equation of motion: $m\dot{\vec{v}} = m\vec{g} - b\vec{v}$. In a Cartesian coordinate system where x is horizontal and y points downward, the vector equation breaks cleanly in to two un-coupled scalar equations: $m\dot{v}_x = 0 - bv_x$, and $m\dot{v}_y = mg - bv_y$. The horizontal and vertical equations can be solved separately, and the full solution can be constructed later.

The horizontal equation can be written as $\dot{v}_x = -\frac{b}{m}v_x = -\frac{1}{\tau}v_x$, where we have defined a characteristic time of $\tau \equiv m/b$. Integrating the equation after separating variables, one finds $v_x(t) = v_{x0}e^{-t/\tau}$, where $v_x(0) = v_{x0}$ is the initial velocity. The velocity simply decays to zero after its initial kick, and the "1/e decay time" is τ. The velocity equation can be integrated to find the position of the particle moving horizontally under the influence of linear drag: $x(t) - x(0) = x_\infty\left(1 - e^{-t/\tau}\right)$, where $x_\infty = v_{x0}\tau$. The particle only moves a finite distance x_∞ before coming to rest.